COLLISION OF JETS EMERGING FROM CHANNELS WITH PARALLEL WALLS

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Much effort in the past several years has gone into the investigation, development, and design of digital computers and devices capable of carrying out logical operations and performing control functions by means of fluid-jet elements. These devices are intended to carry out the same operations as electronic circuits. Jet elements are also used in various types of sensing and operating equipment [1,2].

Jet elements are based on two principal mechanisms: a) the interaction of liquid or gas jets, and b) stream-wall interaction (the so-called Coanda effect). Jet elements usually combine both of these mechanisms [2].

Cronin [3] used conformal mapping to solve the problem of collision of jets emerging from channels with parallel walls for the case of an incompressible fluid with special reference to jet amplifiers. However, he did not obtain analytical expressions in finite form for the required quantities; his computations for an angle of 18° between the channel axes are carried out by numerical integration.

In the present paper we consider the collision of gas jets emerging from channels with parallel walls whose axes form an arbitrary angle. The flow is assumed to be subsonic, planar, steady, potential, and adiabatic. The problem is solved by the method of Chaplygin [4] as generalized by Fal'kovich [5] for the case of several characteristic velocities. This enables us to find expressions for the stream function, the jet compression coefficient, and the geometric flow elements. We extend the solution to the case of an incompressible fluid by a limiting process. Analytical formulas in closed form are obtained for the case where the channel axes are at right angles to each other. These formulas were used as a basis for detailed numerical calculations.

Our results can be used for computing the geometric characteristics of discrete-action jet amplifiers and analog-type jet amplifiers [3].

1. Let us consider the collision of jet streams emerging from channels of finite width and parallel walls. Two of the walls meet at the point O and form the angle $\alpha = \sigma\pi$ $(0 < \sigma \leqslant 1)$ (Fig. 1).

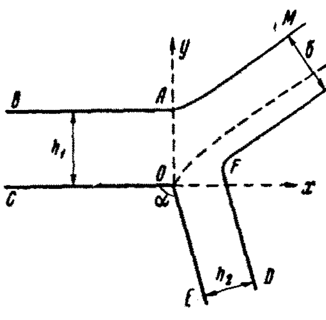

Fig. 1

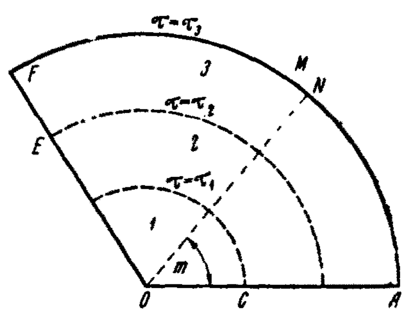

Fig. 2

Here OC, AB, OE, FD are the channel walls; AM, FN are the free jet surfaces; v_1, v_2, v_3 are the gas velocities at the infinitely distant channel cross sections

BC, ED and at the jet cross section MN; ρ_1, ρ_2, ρ_3 are the gas densities at the same cross sections; h_1, h_2, δ are the widths of the channels and of the confluent jet at infinity; m is the angle of inclination of the jet to the x-axis at infinity.

We assume that the flow is planar, subsonic, steady, potential, and adiabatic, and that the gas in both channels is of the same nature. We shall limit ourselves to the case where the jet boundary OL is not a line of discontinuity, but rather a streamline common to the two flows along which the gas parameters vary continuously. The potential solutions for both jets are therefore analytical continuations of each other. This implies, in turn, that an analytical solution for the two flows together can be obtained by the usual method applied to potential flow problems. The criterion of continuity of the flow parameters along the boundary OL in a certain special case is given in [6].

Let us assume that along the streamlines COL and EOL meeting at the point O the stream function $\psi = 0$. If we denote the gas discharge rates at the cross sections BC and ED by Q_1 and Q_2, respectively, and the gas discharge rate at the cross section MN by Q, so that

$$Q = Q_1 + Q_2 \tag{1.1}$$

then the stream function is $\psi = Q_1$ on the streamline BAM and $\psi = -Q_2$ on the streamline DFM.

In the plane of the velocity hodograph with the polar coordinates τ, θ, i.e. in the plane of the variables $\tau = v^2 / v_{\max}^2$ (where v is the velocity, $v_{\max}$ the maximum velocity, and θ the angle of inclination of the velocity to the x-axis (Fig. 1)) the flow domain under consideration can be represented as a circular sector of radius τ_3 and the vertex angle α (Fig. 2). The boundary conditions are

$$\begin{aligned}
\psi &= 0 && \text{for } \theta = 0,\ 0 \leqslant \tau \leqslant \tau_1 \\
\psi &= Q_1 && \text{for } \theta = 0,\ \tau_1 \leqslant \tau \leqslant \tau_3 \\
\psi &= 0 && \text{for } \theta = \sigma\pi,\ 0 \leqslant \tau \leqslant \tau_2 \\
\psi &= -Q_2 && \text{for } \theta = \sigma\pi,\ \tau_2 \leqslant \tau \leqslant \tau_3
\end{aligned} \tag{1.2}$$

$$\begin{aligned}
\psi &= Q_1 && \text{for } \tau = \tau_3,\ 0 \leqslant \theta \leqslant m \\
\psi &= -Q_2 && \text{for } \tau = \tau_3\ m \leqslant \theta \leqslant \sigma\pi
\end{aligned} \tag{1.3}$$

We have thus reduced solution of our problem to finding the solution of the interior Dirichlet problem for Chaplygin's equation

$$4\tau^2(1-\tau)\frac{\partial^2\psi}{\partial\tau^2} + 4\tau[1+(\beta-1)\tau]\frac{\partial\psi}{\partial\tau} + [1-(2\beta+1)\tau]\frac{\partial^2\psi}{\partial\theta^2} = 0 \tag{1.4}$$

in the appropriate subregions of the circular sector. Here $\beta = 1/(\varkappa - 1)$, $\varkappa = C_p / C_v$. Since $\tau < 1/(2\beta+1)$, it follows that Eq. (1.4) is an elliptic equation in the region under consideration.

Following [5], we shall seek the solution of the problem in the form

$$\begin{aligned}
\psi_1(\tau,\theta) &= \sum_{n=1}^{\infty} a_n z_\lambda(\tau)\sin\lambda\theta, \qquad \lambda = \frac{n}{\sigma} \\
\psi_2(\tau,\theta) &= Q_1\frac{\sigma\pi-\theta}{\sigma\pi} + \sum_{n=1}^{\infty}[A_n z_\lambda(\tau) + B_n\zeta_\lambda(\tau)]\sin\lambda\theta \\
\psi_3(\tau,\theta) &= Q_1 - Q\frac{\theta}{\sigma\pi} + \sum_{n=1}^{\infty}[C_n z_\lambda(\tau) + D_n\zeta_\lambda(\tau)]\sin\lambda\theta
\end{aligned} \tag{1.5}$$

Here the subscript of ψ denotes the number of the subregion of the circular sector to which the solution refers; $z_\lambda(\tau)$ is the integral of the equation

$$4\tau^2(1-\tau)Z_\lambda'' + 4\tau[1+(\beta-1)\tau]Z_\lambda' - \lambda^2[1-(2\beta+1)\tau]Z_\lambda = 0 \qquad (1.6)$$

which is regular for $\tau = 0$; $\zeta_\lambda(\tau)$ is the second linearly independent integral of Eq. (1.6) obtained by Lighthill [7] and Cherry [8,9] and first used in the theory of gas jets by Fal'kovich [5]. For the Wronskian of these integrals we have

$$w_\lambda(\tau) = z_\lambda'(\tau)\zeta_\lambda(\tau) - \zeta_\lambda'(\tau)z_\lambda(\tau) = \lambda(1-\tau)^\beta\tau^{-1} \qquad (1.7)$$

The coefficients a_n, $A_n, \ldots, D_n$ must be determined.

The stream functions given by (1.5) satisfy boundary conditions (1.2). Now let us require fulfillment of boundary conditions (1.3) and of the conditions of analytical continuation through the subregion boundaries, i.e.

$$\begin{aligned} \psi_1(\tau_1,\theta) &= \psi_2(\tau_1,\theta), \quad \frac{\partial\psi_1}{\partial\tau} = \frac{\partial\psi_2}{\partial\tau} \quad \text{for} \quad \tau=\tau_1,\ 0\leqslant\theta\leqslant\sigma\pi \\ \psi_2(\tau_2,\theta) &= \psi_3(\tau_2,\theta), \quad \frac{\partial\psi_2}{\partial\tau} = \frac{\partial\psi_3}{\partial\tau} \quad \text{for} \quad \tau=\tau_2,\ 0\leqslant\theta\leqslant\sigma\pi \end{aligned} \qquad (1.8)$$

Conditions (1.3) and (1.8) yield the system of equations

$$\begin{gathered} C_n z_\lambda(\tau_3) + D_n\zeta_\lambda(\tau_3) = -(2Q/n\pi)\cos\lambda m \\ (A_n - a_n)z_\lambda(\tau_1) + B_n\zeta_\lambda(\tau_1) = -2Q_1/n\pi \\ (A_n - a_n)z_\lambda'(\tau_1) + B_n\zeta_\lambda'(\tau_1) = 0 \\ (A_n - C_n)z_\lambda(\tau_2) + (B_n - D_n)\zeta_\lambda(\tau_2) = (-1)^n 2Q_2/n\pi \\ (A_n - C_n)z_\lambda'(\tau_2) + (B_n - D_n)\zeta_\lambda'(\tau_2) = 0 \end{gathered}$$

Solving this system and making use of (1.7), we obtain the coefficients $a_n, A_n, \ldots, D_n$. This determines stream functions (1.5). The final solution of the problem is of the form (1.9)

$$\psi_1(\tau,\theta) = \frac{2Q}{\sigma\pi}\sum_{n=1}^{\infty}\frac{1}{\lambda}f_\lambda(\tau)\sin\lambda\theta, \quad \psi_2(\tau,\theta) = Q_1\frac{\sigma\pi-\theta}{\sigma\pi} + \frac{2Q}{\sigma\pi}\sum_{n=1}^{\infty}\frac{1}{\lambda}g_\lambda(\tau)\sin\lambda\theta$$

$$\psi_3(\tau,\theta) = Q_1 - Q\frac{\theta}{\sigma\pi} + \frac{2Q}{\sigma\pi}\sum_{n=1}^{\infty}\frac{1}{\lambda}\chi_\lambda(\tau)\sin\lambda\theta$$

Here we have used the notation

$$f_\lambda(\tau) = \left[-\cos\lambda m + \varepsilon_1\frac{T_\lambda'(\tau_1,\tau_3)}{w_\lambda(\tau_1)} + (-1)^n\varepsilon_2\frac{T_\lambda'(\tau_2,\tau_3)}{w_\lambda(\tau_2)}\right]\frac{z_\lambda(\tau)}{z_\lambda(\tau_3)}$$

$$g_\lambda(\tau) = \varepsilon_1\frac{z_\lambda'(\tau_1)}{z_\lambda(\tau_3)}\frac{T_\lambda(\tau,\tau_3)}{w_\lambda(\tau_1)} + \left[-\cos\lambda m + (-1)^n\varepsilon_2\frac{T_\lambda'(\tau_2,\tau_3)}{w_\lambda(\tau_2)}\right]\frac{z_\lambda(\tau)}{z_\lambda(\tau_3)}$$

$$\chi_\lambda(\tau) = -\frac{z_\lambda(\tau)}{z_\lambda(\tau_3)}\cos\lambda m + \left[\varepsilon_1\frac{z_\lambda'(\tau_1)}{w_\lambda(\tau_1)} + (-1)^n\varepsilon_2\frac{z_\lambda'(\tau_2)}{w_\lambda(\tau_2)}\right]\frac{T_\lambda(\tau,\tau_3)}{Z_\lambda(\tau_3)} \qquad (1.10)$$

in which[10]

$$T_\lambda(\tau, \tau_3) = z_\lambda(\tau)\zeta_\lambda(\tau_3) - \zeta_\lambda(\tau) z_\lambda(\tau_3), \quad T_\lambda(\tau_i, \tau_i) = 0$$
$$T_\lambda'(\tau_i, \tau_3) = [T_\lambda'(\tau, \tau_3)]_{\tau=\tau_i}, \quad T_\lambda'(\tau_i, \tau_i) = w_\lambda(\tau_i) \quad (i = 1, 2, 3) \tag{1.11}$$

$$\varepsilon_1 = \frac{Q_1}{Q} = \frac{h_1 v_1 (1-\tau_1)^\beta}{\delta v_3 (1-\tau_3)^\beta}, \qquad \varepsilon_2 = \frac{Q_2}{Q} = \frac{h_2 v_2 (1-\tau_2)^\beta}{\delta v_3 (1-\tau_3)^\beta} \tag{1.12}$$

We can readily verify from (1.10) - (1.12) that

$$\chi_\lambda(\tau_3) = -\cos\lambda m$$

$$\chi_\lambda'(\tau_3) = \frac{\lambda}{\tau_3}\Big[-x_\lambda(\tau_3)\cos\lambda m + \frac{h_1}{\delta}\left(\frac{\tau_1}{\tau_3}\right)^{1/2}\frac{z_\lambda(\tau_1)}{z_\lambda(\tau_3)}x_\lambda(\tau_1) + $$
$$+ (-1)^n \frac{h_2}{\delta}\left(\frac{\tau_2}{\tau_3}\right)^{1/2}\frac{z_\lambda(\tau_2)}{z_\lambda(\tau_3)}x_\lambda(\tau_2)\Big] \tag{1.13}$$

The latter expression was obtained by differentiating (1.10), setting $\tau = \tau_3$, applying (1.7), and introducing the Chaplygin functions

$$x_\lambda(\tau) = \frac{\tau z_\lambda'(\tau)}{\lambda z_\lambda(\tau)}$$

From solution (1.9) for $\sigma = 1$ we can obtain Makeev's solution [10] and other special cases mentioned in his paper.

When $h_1 = h_2$, $v_1 = v_2$, $\rho_1 = \rho_2$, $Q_1 = Q_2$ we obtain from (1.9) the solution for the symmetrical case (Den Gan Kho [11]).

2. Let us determine the compression coefficient and angle of deviation of the jet. Since we shall be using the function $\psi_3(\tau, \theta)$ only, we shall simply write $\psi(\tau, \theta)$. Along any jet surface we have

$$dy = 2\tau\frac{(1-\tau)^{-\beta}}{v}\frac{\partial\psi}{\partial\tau}\sin\theta\, d\theta, \qquad dx = 2\tau\frac{(1-\tau)^{-\beta}}{v}\frac{\partial\psi}{\partial\tau}\cos\theta\, d\theta \tag{2.1}$$

Substituting the stream function $\psi(\tau, \theta)$ into (2.1), integrating over θ from 0 to θ, and setting $\tau = \tau_3$, we obtain the parametric equations of the jet contour AM

$$y = h_1 + \frac{4Q}{\sigma\pi}\frac{\tau_3(1-\tau_3)^{-\beta}}{v_3}\sum_{n=1}^{\infty}\frac{\chi_\lambda'(\tau_3)}{2\lambda}\left[\frac{\sin(\lambda-1)\theta}{\lambda-1} - \frac{\sin(\lambda+1)\theta}{\lambda+1}\right] \tag{2.2}$$

$$x = \frac{4Q}{\sigma\pi}\frac{\tau_3(1-\tau_3)^{-\beta}}{v_3}\left\{\sum_{n=1}^{\infty}\frac{\chi_\lambda'(\tau_3)}{\lambda^2-1} - \sum_{n=1}^{\infty}\frac{\chi_\lambda'(\tau_3)}{2\lambda}\left[\frac{\cos(\lambda-1)\theta}{\lambda-1} + \frac{\cos(\lambda+1)\theta}{\lambda+1}\right]\right\}$$

In similar fashion we obtain the equations of the contour FN

$$y' = -h_2\cos\sigma\pi + \frac{4Q}{\sigma\pi}\frac{\tau_3(1-\tau_3)^{-\beta}}{v_3}\times$$
$$\times\left\{\sin\sigma\pi\sum_{n=1}^{\infty}(-1)^n\frac{\chi_\lambda'(\tau_3)}{\lambda^2-1} + \sum_{n=1}^{\infty}\frac{\chi_\lambda'(\tau_3)}{2\lambda}\left[\frac{\sin(\lambda-1)\theta}{\lambda-1} - \frac{\sin(\lambda+1)\theta}{\lambda+1}\right]\right\}$$

$$x' = h_2\sin\sigma\pi + \frac{4Q}{\sigma\pi}\frac{\tau_3(1-\tau_3)^{-\beta}}{v_3}\times \tag{2.3}$$
$$\times\left\{\cos\sigma\pi\sum_{n=1}^{\infty}(-1)^n\frac{\chi_\lambda'(\tau_3)}{\lambda^2-1} - \sum_{n=1}^{\infty}\frac{\chi_\lambda'(\tau_3)}{2\lambda}\left[\frac{\cos(\lambda-1)\theta}{\lambda-1} + \frac{\cos(\lambda+1)\theta}{\lambda+1}\right]\right\}$$

Here x', y' are the coordinates of points on the contour FN.

In order to find the relationships among the parameters of the problem we make use of Zhukovskii's assumption[12] that the points M and N lie on equipotential lines. Under this assumption we have

$$y_M - y_N' = \delta \cos m, \ x_N' - x_M = \delta \sin m \tag{2.4}$$

Recalling (2.2) and (2.3), we transform (2.4) into

$$\delta \cos m = h_1 + h_2 \cos \sigma\pi - 4\delta\tau_3 \frac{\sin \sigma\pi}{\sigma\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{\lambda^2 - 1} \chi_\lambda'(\tau_3) \tag{2.5}$$

$$\delta \sin m = h_2 \sin \sigma\pi + \frac{4\delta\tau_3}{\sigma\pi}\left[\cos \sigma\pi \sum_{n=1}^{\infty} \frac{(-1)^n}{\lambda^2 - 1} \chi_\lambda'(\tau_3) - \sum_{n=1}^{\infty} \frac{\chi_\lambda'(\tau_3)}{\lambda^2 - 1}\right] \tag{2.6}$$

To these equations we add continuity equation (1.1), which we rewrite as

$$\delta v_3 (1 - \tau_3)^\beta = h_1 v_1 (1 - \tau_1)^\beta + h_2 v_2 (1 - \tau_2)^\beta$$

From this we obtain

$$\delta = h_1 \left(\frac{\tau_1}{\tau_3}\right)^{1/2} \left(\frac{1 - \tau_1}{1 - \tau_3}\right)^\beta + h_2 \left(\frac{\tau_2}{\tau_3}\right)^{1/2} \left(\frac{1 - \tau_2}{1 - \tau_3}\right)^\beta \tag{2.7}$$

Dividing (2.6) by (2.5) and recalling (1.13), (2.7), we obtain

$$\begin{aligned} \operatorname{tg} m = \{&\Omega_\lambda(\tau_1, m) - \Pi_\lambda(\tau_1) + \nu[\sin \sigma\pi + \Omega_\lambda(\tau_2, m) + H_\lambda(\tau_2)] + \\ &+ [\Phi_\lambda(\tau_1, m) - H_\lambda(\tau_1) + \nu\Phi_\lambda(\tau_2, m) + \nu\Pi_\lambda(\tau_2)] \cos \sigma\pi\} \times \\ &\times \{1 + \nu \cos \sigma\pi - [\Phi_\lambda(\tau_1, m) - H_\lambda(\tau_1) + \nu\Phi_\lambda(\tau_2, m) + \\ &\nu\Pi_\lambda(\tau_2)] \sin \sigma\pi\}^{-1} \end{aligned} \tag{2.8}$$

Here

$$H_\lambda(\tau) = \frac{4}{\sigma\pi}\left(\frac{\tau}{\tau_3}\right)^{1/2} \sum_{n=1}^{\infty} (-1)^{n-1} \frac{\lambda}{\lambda^2 - 1} \frac{z_\lambda(\tau)}{z_\lambda(\tau_3)} x_\lambda(\tau)$$

$$\Pi_\lambda(\tau) = \frac{4}{\sigma\pi}\left(\frac{\tau}{\tau_3}\right)^{1/2} \sum_{n=1}^{\infty} \frac{\lambda}{\lambda^2 - 1} \frac{z_\lambda(\tau)}{z_\lambda(\tau_3)} x_\lambda(\tau)$$

$$\Phi_\lambda(\tau, m) = \frac{4\Delta(\tau, \tau_3)}{\sigma\pi} \sum_{n=1}^{\infty} (-1)^{n-1} \frac{\lambda}{\lambda^2 - 1} x_\lambda(\tau_3) \cos \lambda m \tag{2.9}$$

$$\Omega_\lambda(\tau, m) = \frac{4\Delta(\tau, \tau_3)}{\sigma\pi} \sum_{n=1}^{\infty} \frac{\lambda}{\lambda^2 - 1} x_\lambda(\tau_3) \cos \lambda m,$$

$$\nu = \frac{h_2}{h_1}, \qquad \Delta(\tau, \tau_3) = \left(\frac{\tau}{\tau_3}\right)^{1/2} \left(\frac{1 - \tau}{1 - \tau_3}\right)^\beta$$

From (2.5) with allowance for (1.13), (2.7), (2.9) we find that

$$\nu = \frac{1 - \Delta(\tau_1, \tau_3) \cos m + [H_\lambda(\tau_1) - \Phi_\lambda(\tau_1, m)] \sin \sigma\pi}{\Delta(\tau_2, \tau_3) \cos m - \cos \sigma\pi + [\Pi_\lambda(\tau_2) + \Phi_\lambda(\tau_2, m)] \sin \sigma\pi} \tag{2.10}$$

By the "compression coefficient" of the confluent asymmetric jet we mean the ratio of the smallest width δ of this jet to the sum $h_1 + h_2$ We then find directly from Eq. (2.7) that

$$k = \frac{\delta}{h_1 + h_2} = \frac{1}{1+\nu}\left(\frac{\tau_1}{\tau_3}\right)^{1/2}\left(\frac{1-\tau_1}{1-\tau_3}\right)^{\beta} + \frac{\nu}{1+\nu}\left(\frac{\tau_2}{\tau_3}\right)^{1/2}\left(\frac{1-\tau_2}{1-\tau_3}\right)^{\beta} \tag{2.11}$$

Relations (2.8), (2.10), and (2.11) give us m, ν, k as functions of τ_1, τ_2, τ_3, $\sigma\pi$. Let us consider some special cases.

Case 1. Let us set $\sigma = 1$, $\lambda = n$. We note that (2.9) yields the equations

$$\lim_{\sigma\to 1}\left[\Omega_\lambda(\tau, m) + \Phi_\lambda(\tau, m)\cos\sigma\pi\right] = \Omega_{2n}(\tau, m)$$

$$\lim_{\sigma\to 1}\left[\Pi_\lambda(\tau) + H_\lambda(\tau)\cos\sigma\pi\right] = \Pi_{2n}(\tau)$$

$$\lim_{\sigma\to 1}\left[H_\lambda(\tau) + \Pi_\lambda(\tau)\cos\sigma\pi\right] = -\Pi_{2n}(\tau)$$

$$\lim_{\sigma\to 1}\Phi_\lambda(\tau, m)\sin\sigma\pi = 2\Delta(\tau, \tau_3)\, x_1(\tau_3)\cos m \tag{2.12}$$

$$\lim_{\sigma\to 1} H_\lambda(\tau)\sin\sigma\pi = 2\left(\frac{\tau}{\tau_3}\right)^{1/2}\frac{z_1(\tau)}{z_1(\tau_3)}\, x_1(\tau)$$

The function $z_1(\tau)$ is elementary [7],

$$z_1(\tau) = \frac{1-(1-\tau)^{\beta+1}}{(\beta+1)\,\tau^{1/2}}$$

We therefore have

$$x_1(\tau) = -\frac{1}{2} + (\beta+1)\frac{\tau(1-\tau)^{\beta}}{1-(1-\tau)^{\beta+1}} \tag{2.13}$$

$$\frac{z_1(\tau)}{z_1(\tau_3)}\, x_1(\tau) = \frac{1}{2}\left(\frac{\tau_3}{\tau}\right)^{1/2}\frac{(1-\tau)^{\beta}\,[1+(2\beta+1)\tau - 1}{1-(1-\tau_3)^{\beta+1}}$$

Recalling (2.12), (2.13), we transform (2.10), (2.8) into

$$\cos m = \frac{1+(2\beta+1)\tau_1}{2(\beta+1)(\tau_1\tau_3)^{1/2}}\left[1 - \nu\frac{1+(2\beta+1)\tau_2}{1+(2\beta+1)\tau_1}\left(\frac{1-\tau_2}{1-\tau_1}\right)^{\beta} - \right.$$
$$\left. - \frac{(1-\nu)(1-\tau_3)}{1+(2\beta+1)\tau_1}\left(\frac{1-\tau_3}{1-\tau_1}\right)^{\beta}\right]\left[1 + \nu\left(\frac{\tau_2}{\tau_1}\right)^{1/2}\left(\frac{1-\tau_2}{1-\tau_1}\right)^{\beta}\right]^{-1} \tag{2.14}$$

$$\nu = \frac{\Omega_{2n}(\tau_1, m) - \Pi_{2n}(\tau_1) - \Delta(\tau_1, \tau_3)\sin m}{\Delta(\tau_2, \tau_3)\sin m + \Pi_{2n}(\tau_2) - \Omega_{2n}(\tau_2, m)} \tag{2.15}$$

respectively.
In this special case Eq. (2.14) follows directly from the familiar Euler theorem.

Case 2. We set $\sigma = 1/2$, $\lambda = 2n$. Here (2.8), (2.10) become

$$\operatorname{tg} m = \frac{\Omega_{2n}(\tau_1, m) - \Pi_{2n}(\tau_1) + \nu\,[1 + \Omega_{2n}(\tau_2, m) + H_{2n}(\tau_2)]}{1 + H_{2n}(\tau_1) - \Phi_{2n}(\tau_1, m) - \nu\,[\Pi_{2n}(\tau_2) + \Phi_{2n}(\tau_2, m)]} \tag{2.16}$$

$$\nu = \frac{1 - \Delta(\tau_1, \tau_3)\cos m + H_{2n}(\tau_1) - \Phi_{2n}(\tau_1, m)}{\Delta(\tau_2, \tau_3)\cos m + \Pi_{2n}(\tau_2) + \Phi_{2n}(\tau_2, m)} \tag{2.17}$$

3. In the case of an incompressible fluid

$$\lim_{v_{max}\to\infty}\frac{z_\lambda(\tau)}{z_\lambda(\tau_3)} = \left(\frac{v}{v_3}\right)^{\lambda}; \qquad \lim_{v_{max}\to\infty} x_\lambda(\tau) = \frac{1}{2}, \quad \lim_{v_{max}\to\infty}\Delta(\tau, \tau_3) = \frac{v}{v_3}$$

and expressions (2.8), (2.10), (2.11) become

$$\operatorname{tg} m = \{\Omega_\lambda^\circ (v_1, m) - \Pi_\lambda^\circ (v_1) + \nu [\sin \sigma\pi + \Omega_\lambda^\circ (v_2, m) + H_\lambda^\circ (v_2)] +$$
$$+ [\Phi_\lambda^\circ (v_1, m) - H_\lambda^\circ (v_1) + \nu \Phi_\lambda^\circ (v_2, m) + \nu \Pi_\lambda^\circ (v_2)] \cos \sigma\pi]\} \times$$

Table

t	v_2/v_1	ν	m	k
		$q=0.20$		
0.55	2.750	0.106	32°04′	0.517
0.60	3.000	1.652	61 05	0.351
0.60	3.000	2.471	64 57	0.315
0.65	3.250	0.812	53 14	0.448
0.70	3.500	0.589	50 00	0.515
0.70	3.500	1.947	62 03	0.370
0.75	3.750	0.455	47 46	0.578
0.75	3.750	1.055	56 55	0.468
0.80	4.000	0.359	45 55	0.642
0.80	4.000	0.716	53 42	0.550
0.85	4.250	0.283	44 10	0.707
0.85	4.250	0.512	50 58	0.630
0.90	4.500	0.219	42 16	0.774
0.90	4.500	0.366	48 11	0.712
		$q=0.30$		
0.55	1.833	0.760	46°34′	0.442
0.55	1.833	3.120	64 16	0.361
0.60	2.000	0.538	42 35	0.495
0.60	2.000	7.529	70 03	0.335
0.65	2.167	0.418	40 15	0.547
0.65	2.167	2.163	58 51	0.411
0.65	2.167	4.456	65 11	0.364
0.70	2.333	0.337	38 33	0.599
0.70	2.333	1.077	51 36	0.493
0.75	2.500	0.275	37 11	0.653
0.75	2.500	0.745	48 04	0.558
0.80	2.667	0.225	35 59	0.708
0.80	2.667	0.549	45 22	0.623
0.85	2.833	0.183	34 48	0.765
0.85	2.833	0.411	42 59	0.690
0.90	3.000	0.145	33 32	0.824
0.90	3.000	0.304	40 33	0.760
		$q=0.40$		
0.45	1.125	0.646	39°28′	0.430
0.45	1.125	1.804	54 45	0.418
0.50	1.250	0.445	35 23	0.469
0.50	1.250	3.260	62 17	0.423
0.55	1.375	0.345	33 17	0.512
0.55	1.375	5.807	67 12	0.422
0.60	1.500	0.279	31 51	0.556
0.65	1.625	0.230	30 50	0.603
0.65	1.625	1.141	47 25	0.517
0.70	1.750	0.192	29 59	0.652
0.70	1.750	0.792	43 37	0.567
0.75	1.875	0.161	29 16	0.701
0.75	1.875	0.592	40 58	0.620
0.80	2.000	0.134	28 36	0.753
0.80	2.000	0.454	38 48	0.675
0.85	2.125	0.111	27 56	0.805
0.85	2.125	0.349	36 49	0.734
0.90	2.250	0.090	27 13	0.859
0.90	2.250	0.263	34 48	0.796
		$q=0.50$		
0.35	0.700	0.443	30°35′	0.396
0.35	0.700	1.400	46 49	0.438
0.40	0.800	0.307	27 43	0.424
0.40	0.800	1.225	54°37′	0.469
0.45	0.900	0.237	26 17	0.460
0.45	0.900	3.388	60 12	0.489
0.50	1.000	0.192	25 22	0.500
0.50	1.000	5.217	63 46	0.500
0.55	1.100	0.159	24 42	0.543
0.55	1.100	8.715	68 17	0.505
0.60	1.200	0.133	24 12	0.588
0.60	1.200	1.252	43 56	0.544
0.60	1.200	6.159	63 11	0.514
0.65	1.300	0.113	23 47	0.635
0.65	1.300	0.864	39 47	0.580
0.70	1.400	0.096	23 26	0.682
0.70	1.400	0.650	37 07	0.621
0.75	1.500	0.082	23 08	0.731
0.75	1.500	0.505	35 01	0.666
0.80	1.600	0.069	22 50	0.781
0.80	1.600	0.397	33 19	0.715
0.85	1.700	0.058	22 32	0.831
0.85	1.700	0.310	31 41	0.767
0.90	1.800	0.047	22 12	0.882
0.90	1.800	0.238	30 02	0.823
		$q=0.60$		
0.20	0.333	0.405	25°03′	0.315
0.20	0.333	0.606	28 55	0.351
0.25	0.417	0.191	21 00	0.306
0.25	0.417	1.265	40 11	0.446
0.30	0.500	0.133	19 57	0.335
0.30	0.500	1.860	47 25	0.495
0.35	0.583	0.102	19 24	0.373
0.35	0.583	2.588	53 15	0.530
0.40	0.667	0.081	19 03	0.415
0.40	0.667	3.580	58 08	0.556
0.45	0.750	0.067	18 49	0.459
0.45	0.750	5.057	62 15	0.575
0.50	0.833	0.056	18 31	0.505
0.50	0.833	7.500	65 48	0.588
0.55	0.917	0.048	18 30	0.552
0.55	0.917	1.424	40 50	0.579
0.55	0.917	4.770	57 54	0.591
0.60	1.000	0.041	18 23	0.600
0.60	1.000	0.970	36 14	0.603
0.60	1.000	9.687	64 48	0.600
0.65	1.083	0.035	18 17	0.648
0.65	1.083	0.734	33 31	0.620
0.70	1.167	0.030	18 12	0.697
0.70	1.167	0.575	31 31	0.664
0.75	1.250	0.026	18 07	0.746
0.75	1.250	0.458	29 53	0.703
0.80	1.333	0.022	18 02	0.796
0.80	1.333	0.366	28 26	0.746
0.85	1.417	0.018	17 57	0.846
0.85	1.417	0.291	27 04	0.794
0.90	1.500	0.015	17 52	0.896
0.90	1.500	0.226	25 41	0.845

Table (continued)

t	v_2/v_1	ν	m	k
		$q = 0.70$		
0.15	0.214	1.200	32°56′	0.450
0.20	0.286	1.698	40 00	0.515
0.25	0.357	2.268	46 08	0.562
0.30	0.428	2.971	51 27	0.599
0.35	0.500	3.896	56 02	0.628
0.40	0.571	5.194	60 01	0.652
0.45	0.643	7.156	63 29	0.669
0.50	0.714	1.659	37 42	0.625
0.50	0.714	4.227	52 41	0.662
0.55	0.786	1.118	32 35	0.629
0.55	0.786	7.755	60 32	0.683
0.60	0.857	0.851	29 48	0.646
0.65	0.928	0.673	27 49	0.670
0.70	1.000	0.542	26 14	0.700
0.75	1.071	0.440	24 54	0.735
0.80	1.143	0.356	23 41	0.774
0.85	1.214	0.286	22 31	0.817
0.90	1.286	0.225	21 18	0.863
		$q = 0.80$		
0.15	0.186	2.171	37°47′	0.595
0.20	0.250	2.789	44 05	0.642
0.25	0.312	3.522	49 27	0.678
0.30	0.375	4.447	54 01	0.708
0.35	0.438	5.684	57 58	0.733
0.40	0.500	7.434	61 24	0.753
0.45	0.562	1.886	33 15	0.679
0.45	0.562	4.380	48 27	0.735
0.50	0.625	1.300	28°08′	0.670
		$q = 0.80$		
0.50	0.625	7.342	56 38	0.764
0.55	0.688	1.003	25 23	0.675
0.60	0.750	0.804	23 27	0.689
0.65	0.812	0.655	21 57	0.709
0.70	0.875	0.538	20 41	0.735
0.75	0.938	0.444	19 34	0.765
0.80	1.000	0.364	18 32	0.800
0.85	1.062	0.296	17 31	0.839
0.90	1.125	0.235	16 28	0.881
		$q = 0.90$		
0.15	0.167	3.724	41 50	0.741
0.20	0.222	4.571	47 44	0.774
0.25	0.278	5.593	52 30	0.801
0.30	0.333	6.899	56 28	0.824
0.35	0.389	8.660	59 50	0.843
0.40	0.444	1.965	24 56	0.732
0.40	0.444	5.781	47 04	0.826
0.45	0.500	1.474	21 09	0.718
0.45	0.500	8.469	54 02	0.852
0.50	0.556	1.176	18 52	0.716
0.55	0.611	0.962	17 12	0.722
0.60	0.667	0.798	15 54	0.733
0.65	0.722	0.667	14 48	0.750
0.70	0.778	0.558	13 50	0.772
0.75	0.834	0.467	12 58	0.798
0.80	0.889	0.389	12 08	0.828
0.85	0.945	0.320	11 19	0.862
0.90	1.000	0.257	10 27	0.900

Formulas (3.3) were used to carry out detailed numerical computations whose results are given in the Table.

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SELF-SIMILAR MOTIONS OF A RADIATION-HEATED GAS BEHIND THE ABSORPTION-INITIATING SHOCK WAVE FRONT

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When acted on by sufficiently powerful light beams, gases which transmit radiation under ordinary conditions can experience breakdown. A survey of experimental data and theoretical information on this phenomenon will be found in [1], one of whose aspects is the shifting of the absorption zone to meet the oncoming light beam.

The present paper concerns the motion of the gas and heating behind the absorption wave propagating solely as a result of one of the mechanisms noted in [1] (the hydrodynamic mechanism). The mechanism consists essentially in the following: a shock wave begins to propagate from the breakdown zone, which is characterized by the intensive release of energy and a considerable increase in pressure. Ionization occurs at the shock wave front, and this makes possible the absorption of radiation as a result of the braking mechanism.

The heating of the gas which results in excitation of the atoms and ions also produces absorption (because of the photoelectric effect from highly excited states). If the gas ahead of the front is cold and nonionized, then it usually transmits radiation in the optical range. Thus, the shock wave front marks the boundary at which radiation absorption begins (i.e. it initiates absorption and energy release due to absorption). There are, of course, other factors which can produce such shock waves (electrical discharges, vaporization of the surface of a solid body under one type of radiation or another, etc.).

Absorption of radiation at small distances from the shock wave front produces a detonation wave [1].

If the radiation flux incident on the shock wave front varies, then the detonation wave propagates with a variable velocity. It is of interest to consider gas motions behind the fronts of such shock waves. With a power law of variation of the radiation flux with time, $q \sim t^\alpha$, the velocity of the detonation wave also varies according to a power law, and the problem is self-similar.

It is also interesting to consider gas motion in cases where the radiation is absorbed at distances comparable with characteristic dimension of the problem, and even at distances such that the radiation passes almost freely through the heated gas behind the shock wave front (through optically thin gas layers).

1. The radiation absorption coefficient $\varkappa$ due to free-free electron transitions in the ionic field depends on the temperature and density in the complete-ionization zone in the following way:

$$\varkappa \sim \varepsilon^{-2} T^{-3/2} \rho \sim \varepsilon^{-2} p^{-3/2} \rho^{5/2}$$

Here ε is the quantum energy, T is the temperature, p is the pressure, and ρ is the density.

In the multiple-ionization zone where absorption also occurs by way of the photoelectric effect from highly excited atomic and ionic states the function $\varkappa(T, \rho)$ can also be approximated by means of a power function,

$$\varkappa = k_q \rho^{-a} p^b = k_q v^a p^b \qquad (1.1)$$